Work

Read from **Lesson 1** of the **Work, Energy and Power** chapter at **The Physics Classroom:**

http://www.physicsclassroom.com/Class/energy/u5l1a.cfm
http://www.physicsclassroom.com/Class/energy/u5l1aa.cfm

MOP Connection: Work and Energy: sublevel 1

1. An **impulse** is a force acting over some amount of time to cause a change in momentum. On the other hand, **work** is a _____**force**_____ acting over some amount of _____**distance**_____ to cause a change in _____**energy**_____.

2. Indicate whether or not the following represent examples of work.

		Work Done?
a.	A teacher applies a force to a wall and becomes exhausted. Explanation: **While there is a force, the force does not cause a displacement. of any object.**	**No**
b.	A weightlifter lifts a barbell above her head. Explanation: **There is a force applied to the barbell and the force causes a displacement of the barbell.**	**Yes**
c.	A waiter carries a tray full of meals across a dining room at a constant speed. Explanation: **There is a upward force applied to the tray but the tray moves horizontally. Upward forces do not cause horizontal displacement.**	**No**
d.	A rolling marble hits a note card and moves it across a table. Explanation: **The marble applies a force to the note card to move it across the table. This is work.**	**Yes**
e.	A shot-putter launches the shot. Explanation: **The shot-putter applies a force to displace the shot from a compressed arm position to an outstretched arm position.**	**Yes**

3. Work is a _____; a + or - sign on a work value indicates information about _____.
    Answer: D
    a. vector; the direction of the work vector
    b. scalar; the direction of the work vector
    c. vector; whether the work adds or removes energy from the object
    d. scalar; whether the work adds or removes energy from the object

4. Which sets of units represent legitimate units for the quantity *work*? Circle all correct answers. Answer: **A, B, C, F**

a. Joule   **b. N x m**

c. Foot x pound   d. kg x m/sec

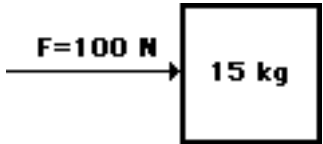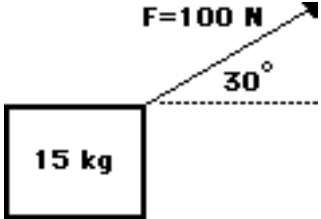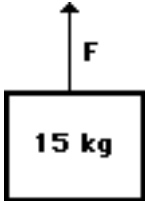e. kg x m/sec^2 **f. kg x m^2/sec^2**

> The amount of work (W) done on an object by a given force can be calculated using the formula
>
> **W = F d cos Θ**
>
> where **F** is the force and **d** is the distance over which the force acts and **Θ** is the angle between **F** and **d**. It is important to recognize that the angle included in the equation is not *just any old angle*; it has a distinct definition that must be remembered when solving such work problems.
>
> **BEWARE**

5. For each situation below, calculate the amount of work done by the applied force. **PSYW**

F=100 N → 15 kg	F=100 N ↗ 30° 15 kg	F ↑ 15 kg
A 100 N force is applied to move a 15 kg object a horizontal distance of 5 meters at constant speed.	A 100 N force is applied at an angle of 30º to the horizontal to move a 15 kg object at a constant speed for a horizontal distance of 5 m.	An upward force is applied to lift a 15 kg object to a height of 5 meters at constant speed.
W = (100 N)•(5 m)•cos(0°) **W = 500 J**	W = (100 N)•(5 m)•cos(30°) **W = 433 J**	The **F** value is equal to m•g since the speed is constant. W = (147 N)•(5 m)•cos(0°) **W = 735 J**

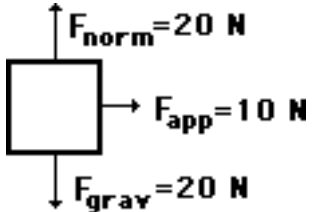6. Indicate whether there is positive (+) or negative (-) work being done on the object.
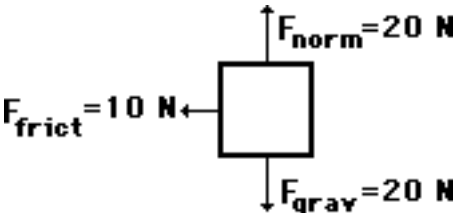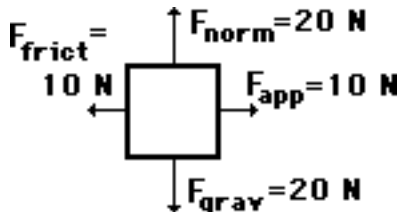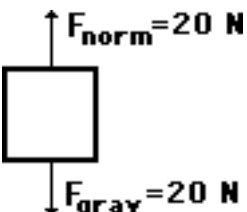
 - a. An eastward-moving **car** skids to a stop across dry pavement.

 + b. A freshman stands on his toes and lifts a **World Civilization book** to the top shelf of his locker.

 + c. At Great America, **a roller coaster car** is lifted to the peak of the first hill on the Shock Wave.

 - d. A catcher puts out his mitt and catches the **baseball**.

 - e. A falling **parachutist** opens the chute and slows down.

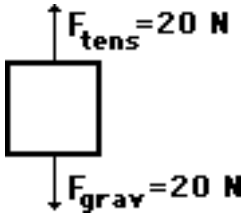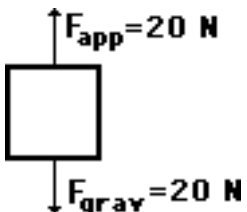**NOTE:** Work is negative when the force and the displacement are in opposite directions.

7. Before beginning its initial descent, a roller coaster car is always pulled up the first hill to a high initial height. Work is done on the car (usually by a chain) to achieve this initial height. A coaster designer is considering three different angles at which to drag the 2000-kg car train to the top of the 60-meter high hill. Her big question is: which angle would require the most work? **All angles result in the same work.** Show your answers and explain. This conclusion is supported by the calculations below. In each case, the angle Θ in the work equation is 0°; this is the angle between the F vector and the displacement (not the incline angle); since these two directions are parallel to each other, the angle is 0°. The work value is just force•distance•cosine(0°). There is little to no difference between the three resulting values. The difference falls outside the level of precision to which the given F and d values have been expressed.

Angle	Force	Distance	Work
35°	$1.15 * 10^4$ N	105 m	$1.21 * 10^6$ J
45°	$1.41 * 10^4$ N	84.9 m	$1.20 * 10^6$ J
55°	$1.64 * 10^4$ N	73.2 m	$1.20 * 10^6$ J

8. The following descriptions and their accompanying free-body diagrams show the forces acting upon an object. For each case, calculate the work done by these forces; use the format of force • displacement • cosine(Θ). Finally, calculate the total work done by all forces.

Free-Body Diagram	Forces Doing Work on the Object Amount of Work Done by Each Force
a. A 10-N force is applied to push a block across a frictionless surface for a displacement of 5.0 m to the right. $F_{norm} = 20$ N $F_{app} = 10$ N $F_{grav} = 20$ N	W_{norm} = (20 N) • (5.0 m) • cos(90°) = **0** J W_{app} = (10 N) • (5.0 m) • cos(0°) = **50** J W_{grav} = (20 N) • (5.0 m) • cos(90°) = **0** J W_{total} = **50** J
b. A 10-N frictional force slows a moving block to a stop along a horizontal surface after a displacement of 5.0 m to the right. $F_{norm} = 20$ N $F_{frict} = 10$ N $F_{grav} = 20$ N	W_{norm} = (20 N) • (5.0 m) • cos(90°) = **0** J W_{grav} = (20 N) • (5.0 m) • cos(90°) = **0** J W_{frict} = (10 N) • (5.0 m) • cos(180°) = **-50** J W_{total} = **-50** J
c. A 10-N forces is applied to push a block across a frictional surface at constant speed for a displacement of 5.0 m to the right. $F_{frict} = 10$ N $F_{norm} = 20$ N $F_{app} = 10$ N $F_{grav} = 20$ N	W_{norm} = (20 N) • (5.0 m) • cos(90°) = **0** J W_{app} = (10 N) • (5.0 m) • cos(0°) = **50** J W_{grav} = (20 N) • (5.0 m) • cos(90°) = **0** J W_{frict} = (10 N) • (5.0 m) • cos(180°) = **-50** J W_{total} = **0** J
d. A 2-kg object is sliding at constant speed across a frictionless surface for a displacement of 5.0 m to the right. $F_{norm} = 20$ N $F_{grav} = 20$ N	W_{norm} = (20 N) • (5.0 m) • cos(90°) = **0** J W_{grav} = (20 N) • (5.0 m) • cos(90°) = **0** J W_{total} = **0** J

Free-Body Diagram	**Forces Doing Work on the Object** **Amount of Work Done by Each Force**
e. A 2-kg object is pulled upward at constant speed by a 20-N force for a vertical displacement of 5.0 m. F_{tens}=20 N [box] F_{grav}=20 N	W_{tens} = (20 N) • (5.0 m) • cos(0°) = **50** J W_{grav} = (20 N) • (5.0 m) • cos(180°) = **-50** J W_{total} = **0** J
f. A 2-kg tray of dinner plates is held in the air and carried a distance of 5.0 m to the right. F_{app}=20 N [box] F_{grav}=20 N	W_{app} = (20 N) • (5.0 m) • cos(90°) = **0** J W_{grav} = (20 N) • (5.0 m) • cos(90°) = **0** J W_{total} = **0** J

9. When a force is applied to do work on an object, does the object always accelerate? __**No**__ Explain why or why not.

Consider some of the examples above - for instance, part c or part e. In both of these examples there is a force doing work. Yet there is an opposite force to balance it and prevent the acceleration. The opposing force also does work so that the net work is zero.

10. Determine the work done in the following situations.
    a. Jim Neysweeper is applying a 21.6-N force downward at an angle of 57.2° with the horizontal to displace a broom a distance of 6.28 m.

    W = F•d•cosine(Θ) = (21.6 N)•(6.28 m)•cosine(57.2°)

    W = **73.5 J**

    b. Ben Pumpiniron applies an upward force to lift a 129-kg barbell to a height of 1.98 m at a constant speed.

    The applied force value is equal to the force of gravity; this value is m•g or 1264.2 N.

    W = F•d•cosine(Θ) = (1264.2 N)•(1.98 m)•cosine(0°)

    W = **$2.50•10^3$ J** (2503.11 … J)

    c. An elevator lifts 12 occupants up 21 floors (76.8 meters) at a constant speed. The average mass of the occupants is 62.8 kg.

    The force that must be applied to lift 12 occupants at a constant speed is equivalent to the weight (m•g) of the occupants; this value is 12•(62.8 kg)•(9.8 N/kg) or 7385.28 N.

    W = F•d•cosine(Θ) = (7385.28 N)•(76.8 m)•cosine(0°)

    W = **$5.67•10^5$ J** (567189.50 … J)

Power

Read from **Lesson 1** of the **Work, Energy and Power** chapter at **The Physics Classroom:**

http://www.physicsclassroom.com/Class/energy/u5l1e.cfm

MOP Connection: Work and Energy: sublevel 2

Review:

1.   A force acting upon an object to cause a displacement is known as ___**work**___.
      a. energy   b. potential   c. kinetic   **d. work**

2.   Two acceptable units for work are __**joule   or   newton•meter**__. Choose two.
      a. joule   b. newton   c. watt   **d. newton•meter**

Power as a Rate Quantity:

3.   Power is defined as the __**the rate at which work**__ is done.
      a. amount of work which   b. direction at which work
      c. angle at which work   **d. the rate at which work**

4.   Two machines (e.g., elevators) might do identical jobs (e.g., lift 10 passengers three floors) and yet the machines might have different power outputs. Explain how this can be so.

      Power is the rate at which work is done. The two machines are doing the same amount of work, yet doing the work at different rates. One machine might do the job very quickly and the other very slowly. The machine that does the work in the least amount of time (quickly) is most powerful.

5.   There are a variety of units for power. Which of the following would be *fitting* units of power (though perhaps not standard)? Include all that apply.   Allowed answers: **A, C, and D**
      a. Watt   b. Joule   **c. Joule / second**   **d. hp**

6.   Two physics students, Will N. Andable and Ben Pumpiniron, are in the weightlifting room. Will lifts the 100-pound barbell over his head 10 times in one minute; Ben lifts the 100-pound barbell over his head 10 times in 10 seconds. Which student does the most work? __**They do the same work**__ Which student delivers the most power? __**Ben is most *powerful***__ Explain your answers.

      The two students are doing the same task - thus, the same work. Yet Ben does this work at a much higher rate; as such, Ben is most powerful.

7.   During the Powerhouse lab, Jack and Jill ran up the hill. Jack is twice as massive as Jill; yet Jill ascended the same distance in half the time. Who did the most work? ___**Jack does more work**___ Who delivered the most power? ___**Their power is the same.**___ Explain your answers.

      These two students are not doing the same task - their work is not the same. Jack must lift twice the mass as Jill; so Jack does twice the work - maybe 1000 J as opposed to Jill's 500 J (just as an example). But since Jill does one-half the work in one-half the time (perhaps 2.0 seconds as opposed to Jack's 4.0 seconds), the ratio of work to time ends up being the same for each. Do the math - it's 250 Watts for each student.

8.   An often-used equation for power is

      Power = force x velocity

      Express an understanding of the meaning of this equation by using it to explain what type of individuals would be the best choice for lineman on a football team.

      When you own your first NFL football team, select players who are strong (exert large forces) and fast (have high velocities). According to this physics equation, these players will be the most powerful linemen. Now that's *physics for better living!*

Work and Power Calculations

Read from **Lesson 1** of the **Work, Energy and Power** chapter at **The Physics Classroom:**

http://www.physicsclassroom.com/Class/energy/u5l1aa.cfm
http://www.physicsclassroom.com/Class/energy/u5l1e.cfm

MOP Connection: Work and Energy: sublevels 1 and 2

1. Bart runs up a 2.91-meter high flight of stairs at a constant speed in 2.15 seconds. If Bart's mass is 65.9 kg, determine the work which he did and his power rating. **PSYW**

    Work = F•d•cosine(Θ) = m•g•d•cosine(Θ) = (65.9 kg)•(9.8 N/kg)•(2.91 m)•cosine(0°)
    Work = 1879.3 … J = **$1.88•10^3$ J**

    Power = W/t = (1879.3 … J) / (2.15 s)
    Power = **874 W**

2. On a recent adventure trip, Anita Break went rock-climbing. Anita was able to steadily lift her 80.0-kg body 20.0 meters in 100 seconds. Determine Anita 's power rating during this portion of the climb. **PSYW**

    Anita needs to apply a force that is equal to her weight in order to elevate her body a distance of 20.0 m. This force is equal to (80 kg)•(9.8 N/kg) = 784 N. So the work she does to lift her body 20.0 m is (784 N)•(20.0 m)•cosine(0°) = 15680 J. Therefore, …

    Power = W/t = (15680 J)/(100 s) = **~157 W**   (156.80 W)

3. A physics teacher owns a family of squirrels. The squirrels have been trained to do push-ups in repetitive fashion. Being connected to an electrical generator, their ongoing exercise is used to help power the home. There are 23 squirrels in the family and their average mass is 1.1 kg.

    They do work on the "up" part of the push-up, raising their body an average distance of 5.0 cm. If the squirrels averages 71 push-ups per minute, then determine the total amount of work done in one minute and the power generated by their activity. **PSYW**

    For one average squirrel doing one average push-up (not that these squirrels are average; they're actually quite extraordinary), the amount of work is F•d•cosine(0°) = m•g•d•cosine(0°). So

    $W_{1 \text{ squirrel, 1 push-up}}$ = (1.1 kg)•(9.8 N/kg)•(0.050 m) = 0.539 J

    Now multiply by 23 since there are 23 squirrels doing these push-ups; and multiply by 71 since there is an average of 71 push-ups per squirrel in a minute. The result is the work done in 1 minute:

    $W_{23 \text{ squirrels, 71 push-up}}$ = (0.539 J)•23•71 = 880. 18 … J = **~880 J**

    Now to determine the power, divide the work by the time.

    P = W/t = (880. 18 … J)/(60 s) = **~15 Watts** (14.669 … W)

4. An elevator motor lifts 715 kg of mass to the height of the fourth floor of an office building (11.0 meters above ground level) at a constant speed in 9.35 seconds. Determine the power rating of the motor. **PSYW**

    The motor must provide an amount of force equal to the weight of 715 kg; this is a force of (715 kg)•(9.8 N/kg) = 7007 N. The work done by this force is F•d•cosine(0°) = (7007 N)•(11.0 m) = 77077 J. Therefore, …

    Power = W/t = (77,077 J)/(9.35 s) = **~$8.24•10^3$ W** (8,243.52 … W)

Energy

Read from **Lesson 1** of the **Work, Energy and Power** chapter at **The Physics Classroom:**

http://www.physicsclassroom.com/Class/energy/u5l1b.cfm
http://www.physicsclassroom.com/Class/energy/u5l1c.cfm
http://www.physicsclassroom.com/Class/energy/u5l1d.cfm

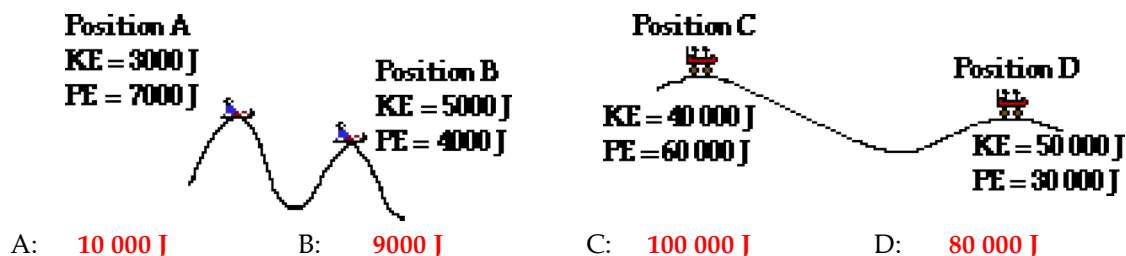**MOP Connection:**   Work and Energy: sublevels 3 and 4

1. Read each of the following statements and identify them as having to do with kinetic energy (KE), potential energy (PE) or both (B).

KE, PE or B?	Statement:
KE	1. If an object is at rest, it certainly does NOT possess this form of energy.
PE	2. Depends upon object mass and object height.
KE	3. The energy an object possesses due to its motion.
B	4. The amount is expressed using the unit joule (abbreviated J).
PE	5. The energy stored in an object due to its position (or height).
PE	6. The amount depends upon the arbitrarily assigned *zero level*.
KE	7. Depends upon object mass and object speed.
B	8. If an object is at rest on the ground (zero height), it certainly does NOT possess this form of energy.

2. A toy car is moving along with 0.40 joules of kinetic energy. If its speed is doubled, then its new kinetic energy will be **1.60 J**.
    a. 0.10 J   b. 0.20 J   c. 0.80 J   **d. 1.60 J**   e. still 0.40 J

3. A young boy's glider is soaring through the air, possessing 0.80 joules of potential energy. If its speed is doubled and its height is doubled, then the new potential energy will be **1.60 J**.
    a. 0.20 J   b. 0.40 J   **c. 1.60 J**   d. 3.20 J   e. still 0.80 J

4. Which would ALWAYS be true of an object possessing a kinetic energy of 0 joules? Answer: **B**
    a. It is on the ground.   **b. It is at rest.**   c. It is moving on the ground
    d. It is moving.   e. It is accelerating.   f. It is at rest above ground level
    g. It is above the ground.   h. It is moving above ground level.

5. Which would ALWAYS be true of an object possessing a potential energy of 0 joules? Answer: **A**
    a. It is on the ground.   b. It is at rest.   c. It is moving on the ground
    d. It is moving.   e. It is accelerating.   f. It is at rest above ground level
    g. It is above the ground.   h. It is moving above ground level.

6. Calculate the kinetic energy of a 5.2 kg object moving at 2.4 m/s. **PSYW**

    $KE = \frac{1}{2} \cdot m \cdot v^2 = \frac{1}{2} \cdot (5.2 \text{ kg}) \cdot (2.4 \text{ m/s})^2 = 14.976 \text{ J} = \underline{\textbf{~15 J}}$

7. Calculate the potential energy of a 5.2 kg object positioned 5.8 m above the ground. **PSYW**

    $PE - m \cdot g \cdot h = (5.2 \text{ kg}) \cdot (9.8 \text{ N/kg}) \cdot (5.8 \text{ m}) = 295.568 \text{ J} = \underline{\textbf{~3.0x10}^2 \textbf{ J}}$

8. Calculate the speed of a 5.2 kg object that possesses 26.1 J of kinetic energy. **PSYW**

    Since $KE = \frac{1}{2} \cdot m \cdot v^2$, it is true that $v = SQRT(2 \cdot KE/m)$. By substitution,

    $v = SQRT((2 \cdot 26.1 \text{ J})/(5.2 \text{ kg})) = \underline{\textbf{~3.2 m/s}}$ (3.168 ... m/s)

    NOTE: SQRT means "square root of".

9. The total mechanical energy of an object is the _____.   Answer: **D**
    a. KE minus the PE of the object   b. PE minus the KE of the object
    c. the initial KE plus the initial PE of the object
    d. KE plus the PE of the object at any instant during its motion
    e. final amount of KE and PE minus the initial amount of KE and PE

10. If an object moves in such a manner as to conserve its total mechanical energy, then ____. Answer: **D**
    a. the amount of kinetic energy remains the same throughout its motion
    b. the amount of potential energy remains the same throughout its motion
    c. the amount of both the kinetic and the potential energy remains the same throughout its motion
    d. the sum of the kinetic energy and the potential energy remains the same throughout its motion

11. Determine the total mechanical energy (TME) of the objects at positions A, B, C and D.

Position A
KE = 3000 J
PE = 7000 J

Position B
KE = 5000 J
PE = 4000 J

Position C
KE = 40 000 J
PE = 60 000 J

Position D
KE = 50 000 J
PE = 30 000 J

   A: **10 000 J**   B: **9000 J**   C: **100 000 J**   D: **80 000 J**

12. Calculate the total mechanical energy (TME) of a 5.2 kg object moving at 2.4 m/s and positioned 5.8 m above the ground. **PSYW**

    $KE = \frac{1}{2} \cdot m \cdot v^2 = \frac{1}{2} \cdot (5.2 \text{ kg}) \cdot (2.4 \text{ m/s})^2 = 14.976 \text{ J}$

    $PE - m \cdot g \cdot h = (5.2 \text{ kg}) \cdot (9.8 \text{ N/kg}) \cdot (5.8 \text{ m}) = 295.568 \text{ J}$

    $TME = KE + PE = 14.976 \text{ J} + 295.568 \text{ J} = \underline{\textbf{~311 J}}$

13. Read the following descriptions and indicate whether the objects' KE, PE and TME increases, decreases or remains the same (=). If it is impossible to tell, then answer ???.

    a. A marble begins at an elevated position on top of an inclined ruler and rolls down to the bottom of the ruler.
       KE: increases   **PE: decreases**   **TME: remains the same**

    b. A marble is rolling along a level table when it hits a note card and slides to a stop.
       KE: decreases   **PE: remains the same**   **TME: decreases**

    c. A cart is pulled from the bottom of an incline to the top of the incline at a constant speed.
       KE: remains the same   **PE: increases**   **TME: increases**

    d. A physics student runs up a staircase at a constant speed.
       KE: remains the same   **PE: increases**   **TME: increases**

    e. A force is applied to a root beer mug to accelerate it from rest across a level countertop.
       KE: increases   **PE: remains the same**   **TME: increases**

    f. A pendulum bob is released from rest from an elevated position and swings to its lowest point.
       KE: increases   **PE: decreases**   **TME: remains the same**

    g. A car skids from a high speed to a stopping position along a level highway.
       KE: decreases   **PE: remains the same**   **TME: decreases**

Work-Energy Relationships

Read from **Lesson 2** of the **Work, Energy and Power** chapter at **The Physics Classroom:**

http://www.physicsclassroom.com/Class/energy/u5l2a.cfm

MOP Connection: Work and Energy: sublevel 5

Important Background: As an object moves, either its total mechanical energy is conserved or mechanical energy is transferred to non-mechanical forms (such as thermal energy, light energy, electrical energy, etc.). Whether there is an energy transfer or an energy conservation depends on whether or not external (a.k.a. non-conservative) forces are doing work. If external forces (or non-conservative forces) are doing work, then the total mechanical energy of the object is not conserved - energy is transferred between mechanical and non-mechanical forms. On the other hand, if external forces do not do work, the total mechanical energy of the object is conserved.

1. Categorize the following force types as being either internal or external forces: F_{grav}; F_{norm}; F_{frict}; F_{air}; F_{app}; F_{tens}; and F_{spring}.

Internal Forces	External Forces			
F_{grav} F_{spring}	F_{norm}	F_{frict}	F_{air}	F_{app} F_{tens}

2. Identify the following as being either always true (**AT**), never true (**NT**) or might be true (**MBT**).

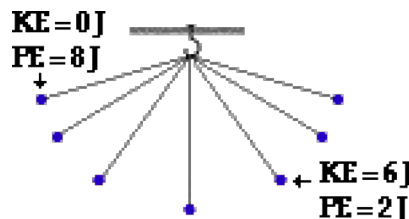| AT, NT, MBT?	Statement:
MBT	a. If gravity does work upon an object, then its total mechanical energy (TME) is conserved. Explanation: it is possible that external (non-conservative) forces are also doing work.
AT	b. If gravity is the only force doing work upon an object, then its total mechanical energy (TME) is conserved.
MBT	c. If a normal force acts upon an object, then its TME will change. Explanation: it is not enough for the normal force to just *act upon* the object; to change the TME, it is necessary that the normal force do work.
MBT	d. If sliding friction does work upon an object, then its TME will decrease. Explanation: there may be an opposing force to friction that does + work.
MBT	e. If only external forces are doing work upon an object, then its TME will be conserved. Explanation: the external forces must be doing net work.
NT	f. If both internal and external forces are doing net work upon an object, then more information is needed to tell if its TME will be conserved. Explanation: if external forces are doing net work, it is a guarantee that the total mechanical energy will be conserved; more info is not needed.
AT	g. If a quantity such as the total mechanical energy is conserved, then that means that it does not change over the course of a motion.

3. Consider the three situations below. Identify whether or not the total mechanical energy (TME) is being conserved. Then indicate if external forces (non-conservative) are doing work.

TME Conserved? ____**Yes**____   TME Conserved? _____**No**_____   TME Conserved? ____**Yes**____

Ext. forces doing work? _**No**_   Ext. forces doing work? _**Yes**_   Ext. forces doing work? _**No**_

4. For each statement, identify which forces (F_{grav}; F_{norm}; F_{frict}; F_{air}; F_{app}; F_{tens}; and F_{spring}) are doing work. Then state whether the total mechanical energy will be conserved.

a. A bungee jumper rapidly decelerates as he reaches the end of his spring-like bungee chord. Ignore the effect of air resistance.	b. A girl releases a softball from rest from a height of 2 meters above the ground; the ball free-falls to the ground.
Forces doing work? **F$_{spring}$, F$_{grav}$**	Forces doing work? **F$_{grav}$**
TME Conserved? **Yes**	TME Conserved? **Yes**
c. A weightlifter briskly raises a 200-pound barbell above his head.	d. A swimmer pushes off the blocks to accelerate forward at the beginning of a race.
Forces doing work? **F$_{app}$, F$_{grav}$**	Forces doing work? **F$_{app}$, F$_{grav}$**
TME Conserved? **No**	TME Conserved? **No**

For questions #5-#13, a physical situation is described. For each situation determine whether the total mechanical energy (TME) of the object (in **bold-face text**) is conserved, increases, or decreases.

NOTE: If non-conservative forces are doing *positive* work, then the total mechanical energy of the object increases. If non-conservative forces are doing *negative* work, then the total mechanical energy of the object decreases. If non-conservative forces are not doing work, then the total mechanical energy is conserved. Answers can also be arrived at on occasion by thinking about changes (increases or decreases) in potential and kinetic energy.

B 5. A force is applied to a **root beer mug** to accelerate it across a level counter-top.
  a. TME conserved b. TME increases c. TME decreases

B 6. A force is applied to a **cart** to raise it up an inclined plane at constant speed.
  a. TME conserved b. TME increases c. TME decreases

A 7. A **marble** starts from rest and rolls down an inclined plane. Ignore friction.
  a. TME conserved b. TME increases c. TME decreases

B 8. A **physics student** runs up a flight of stairs at constant speed.
  a. TME conserved b. TME increases c. TME decreases

A 9. A **baseball** makes its flight through the air. (Neglect F_{air}.)
  a. TME conserved b. TME increases c. TME decreases

C 10. A **coffee filter** is released from rest and falls with a terminal velocity.
  a. TME conserved b. TME increases c. TME decreases

C 11. A **car** skids to a stop while traveling down a steep hill.
  a. TME conserved b. TME increases c. TME decreases

A 12. A **pendulum bob** is tied to a string and swings back and forth. (Neglect F_{air}.)
  a. TME conserved b. TME increases c. TME decreases

C 13. A **marble** hits a note card and slides to a stop.
  a. TME conserved b. TME increases c. TME decreases

Work-Energy Bar Charts

Read from **Lesson 2** of the **Work, Energy and Power** chapter at **The Physics Classroom:**

http://www.physicsclassroom.com/Class/energy/u5l2c.cfm

**MOP Connection:**   Work and Energy: sublevel 6

The work-energy relationship is the most important relationship of the unit. The work done by external forces (W_{ext}) is related to the total mechanical energy of the initial (TME_i) and of the total energy of the final state (TME_f) of a system as follows:

$$TME_i + W_{ext} = TME_f$$

Your goal should be to combine your understanding of kinetic energy, potential energy, and work with the above equation in order to analyze physical situations involving energy changes and transformations and to solve computational problems involving work and energy. One tool that will assist in the analysis of physical situations is a work-energy bar chart. A work-energy bar chart represents the amount of energy present in a system by means of a vertical bar. The length of a bar is representative of the amount of energy present; a longer bar representing a greater amount of energy. According to the work-energy theorem, the initial mechanical energy (kinetic and potential) plus the work done on the system by external forces equals the final mechanical energy (kinetic and potential). Consequently, the sum of the bar heights for any initial condition must equal the sum of the bar heights for the final condition.

Complete the following work-energy bar charts based on the given statement. Then cross out or cancel any terms in the work-energy equation that are either zero or the same on each side.

1.   A ball falls from the top of a pillar to the ground below. The initial state is the ball at rest at the top of the pillar and the final state is the ball just prior to striking the ground. Ignore F_{air}.
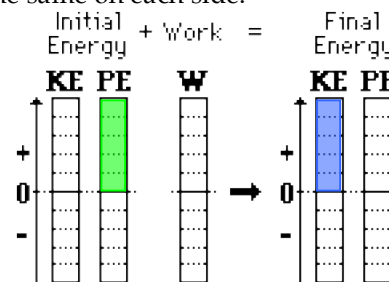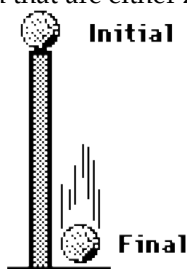
$$\cancel{KE_i} + PE_i + \cancel{W_{ext}} = KE_f + \cancel{PE_f}$$

2.   A car skids from a high speed to a stop with its brakes applied. The initial state is the car traveling at a high speed and the final state is the car at rest. The force of friction does work on the car, thus changing the total mechanical energy.
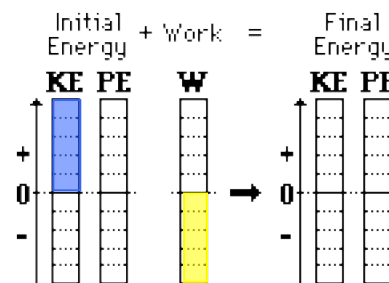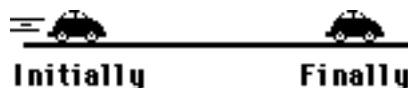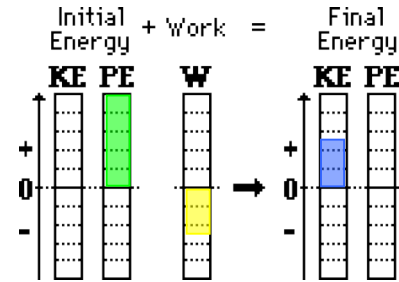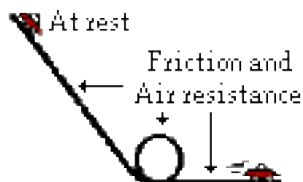

$$KE_i + \cancel{PE_i} + W_{ext} = \cancel{KE_f} + \cancel{PE_f}$$

3. A skier starts from rest on top of hill A and skis into the valley and back up onto hill B. The skier utilizes her poles to propel herself across the snow, thus doing work to change her total mechanical energy. The initial state is on top of hill A and the final state is on top of hill B. Ignore frictional forces.
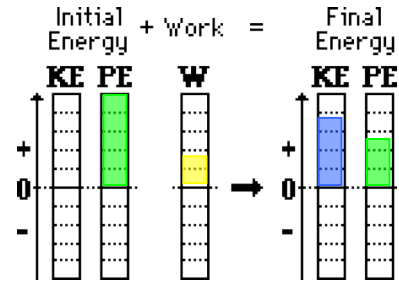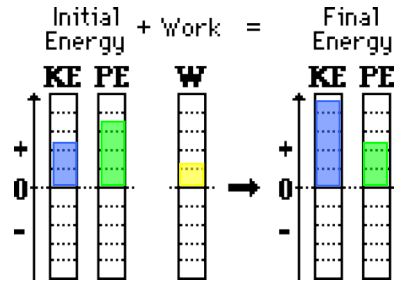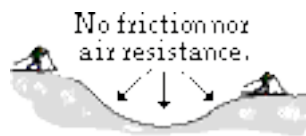
KE_i + PE_i + W_{ext} = KE_f + PE_f

4. A Hot Wheels car starts from rest on top of an inclined plane and rolls down the incline through a loop and along a horizontal surface. The initial state is the car at rest on top of the hill and the final state is the car in motion at the bottom of the hill. Friction and air resistance have a significant effect on the car.
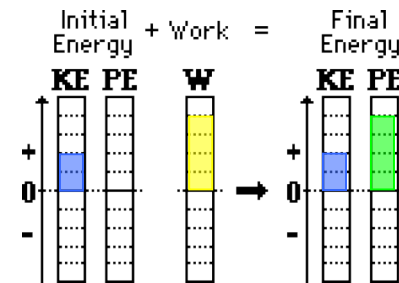
KE_i + PE_i + W_{ext} = KE_f + PE_f

5. A moving cross-country skier skis from the top of a hill down into a valley and up a second smaller hill. The initial state is the skier in motion on top of the first hill and the final state is the skier in motion on top the second hill. He uses his poles to propel himself. Ignore the effect of friction and air resistance.

KE_i + PE_i + W_{ext} = KE_f + PE_f

6. Ben Laborin applies a force to push a crate from the bottom of an inclined plane to the top at a constant speed. The initial state is the crate in motion at the bottom of the hill and the final state is the crate in motion at the top of the hill. Ignore frictional effects.

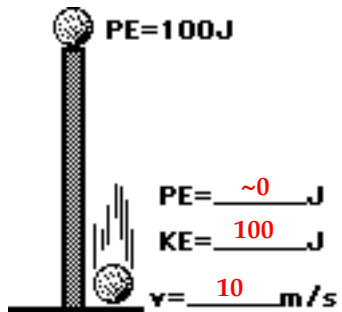KE_i + PE_i + W_{ext} = KE_f + PE_f

Energy Concepts

Read from **Lesson 2** of the **Work, Energy and Power** chapter at **The Physics Classroom:**

http://www.physicsclassroom.com/Class/energy/u5l2b.cfm
http://www.physicsclassroom.com/Class/energy/u5l2bb.cfm
http://www.physicsclassroom.com/Class/energy/u5l2bc.cfm

MOP Connection: Work and Energy: sublevel 7, 8, 9 and 10

1. Consider the falling motion of the ball in the following two <u>frictionless</u> situations. For each situation, indicate the forces doing work upon the ball. Indicate whether the energy of the ball is conserved and explain why. Finally, simplify the work-energy equation and use it to find the kinetic energy and the velocity of the 2-kg ball just prior to striking the ground.
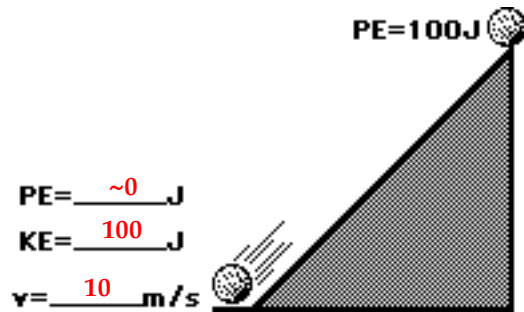
PE=100J   PE=100J

PE=___~0___ J   PE=___~0___ J
KE=___100___ J   KE=___100___ J
v=___10___ m/s   v=___10___ m/s

Forces doing work?   Forces doing work?
Gravity   **Gravity**

TME Conserved: **Yes**   TME Conserved: **Yes**

Explanation: <u>If work is **not** done by non-conservative forces, then TME is conserved.</u>

Explanation: <u>If work is **not** done by non-conservative forces, then TME is conserved.</u>

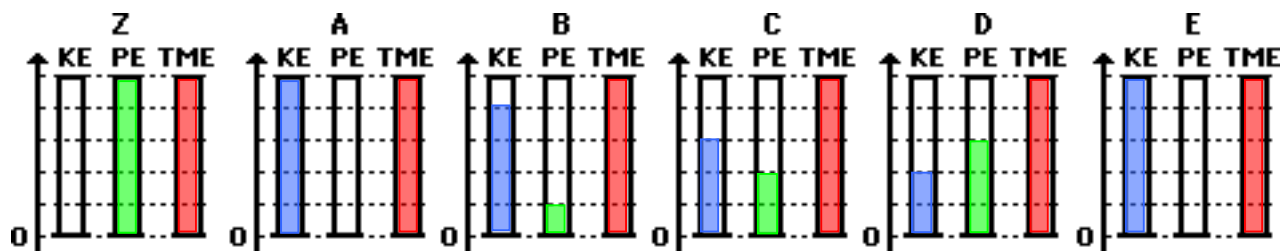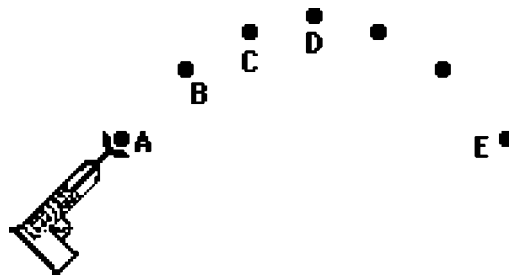$\cancel{KE_i} + PE_i + \cancel{W_{ext}} = KE_f + \cancel{PE_f}$ $\cancel{KE_i} + PE_i + \cancel{W_{ext}} = KE_f + \cancel{PE_f}$

2. Use the work-energy relationship to fill in the blanks for the following system (m=2 kg). Neglect frictional forces. Finally, darken in the bars of the bar chart in order to demonstrate the amount of kinetic energy (KE), potential energy (PE) and total mechanical energy (TME).

PE=10J   PE=2J   PE=0J   PE=___10___ J
KE=0J   KE=___8___ J   KE=___10___ J   KE=___0___ J
h=___0.51___ m   h=___0.10___ m   h=___0___ m   h=___0.51___ m

v=0 m/s   v=___2.8___ m/s   v=___3.2___ m/s   v=0 m/s

3. A dart is launched from a dart gun and subsequently follows a parabolic path typical of any projectile. Five positions in the trajectory of the dart are marked and labeled in the diagram at the right. For each of the five positions and for position Z, fill in the work-energy bar chart in the space below.

   Z: position of dart when springs are compressed.
   A: position of dart after release from springs.

NOTE: The above bar arbitrarily assigns the zero height position to be position A (when the dart has just been released from the springs). At this point, there is neither elastic nor gravitational potential energy. All the mechanical energy is in the form of kinetic energy. As the dart rises along the trajectory to its peak, KE decreases and gravitational PE increases. At the peak, there is still some KE due to the dart's horizontal motion. Falling from the peak, gravitational PE decreases and KE increases. When the dart returns to the zero height (position E), there is no gravitational PE.

4. A 2-kg ball moving at 2 m/s is rolling towards an inclined plane. It eventually rolls up the hill to a position near the top where it momentarily stops prior to rolling back down he incline. Assume negligible friction and air resistance. Construct an energy bar chart for the ball.
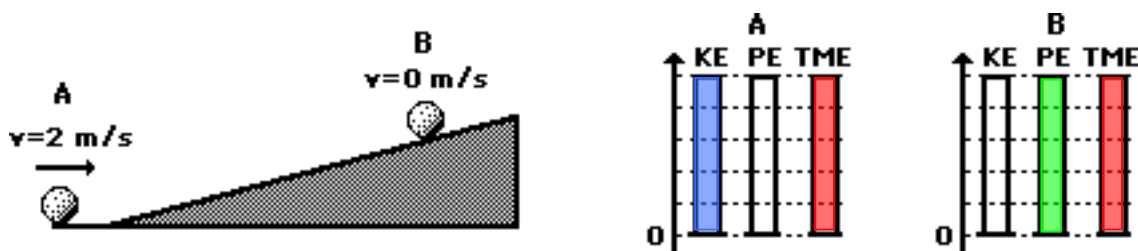
   Simplify the equation below by canceling terms that are either zero or constant. Then use the equation to determine the height to which the ball rises along the incline before stopping.
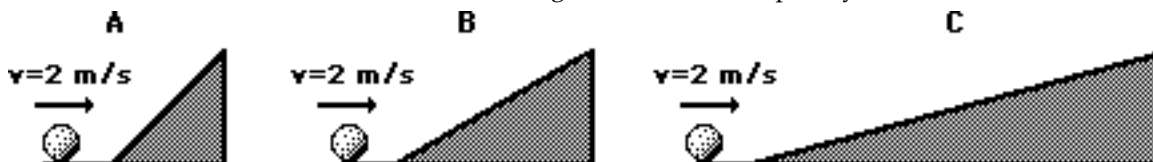
   $$\frac{1}{2} \bullet m \bullet v_i^2 \;+\; \cancel{m \bullet g \bullet h_i} \;+\; \cancel{F \bullet d \bullet \cos\theta} \;=\; \cancel{\frac{1}{2} \bullet m \bullet v_f^2} \;+\; m \bullet g \bullet h_f$$

   ½ • (2 kg) • (2 m/s)² = (2 kg) • (9.8 N/kg) • h
   4 J = (19.6 N) • h
   h = (4 J) / (19.6 N) = **0.20 m**

5. Three identical balls approach three different "frictionless" hills with a speed of 2 m/s. In which case - A, B, or C, (or a tie) - will the ball roll the highest? __**Tie**__ Explain your answer.


   In each case, the balls start with the same amount of total mechanical energy (TME). Since this energy is conserved on a frictionless incline, each ball will also finish with the same TME. At their highest point, the KE of each of the balls is 0 J. Thus, their PE must be the same at that point. Given that each ball has the same mass (*identical*), their final height must be the same. It's a tie!

6.   Fill in the blanks in the following sentence:

> An object starts from rest with a potential energy of 600 J and free-falls towards the ground. After it has fallen to a height of one-fourth of its original height, its total mechanical energy is ___600___ J, its potential energy is ___150___ J, and its kinetic energy is ___450___ J.

Consider the diagram at the right in answering the next three questions. Five locations along a roller coaster track are shown. Assume that there are negligible friction and air resistance forces acting upon the coaster car.
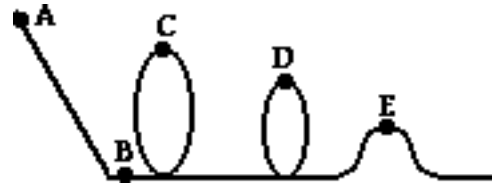
7.   Rank the five locations in order of increasing TME (smallest to largest TME). Use < and or = signs between the blanks.
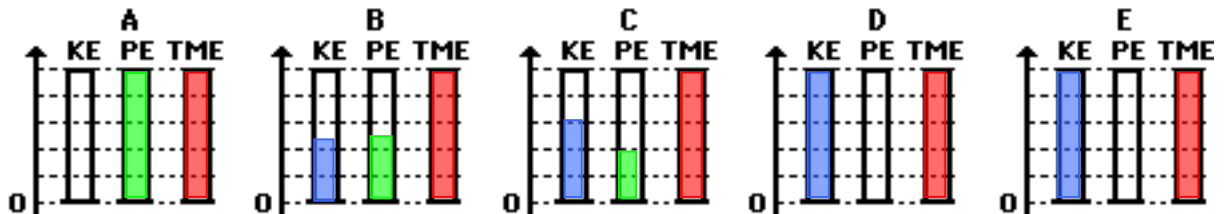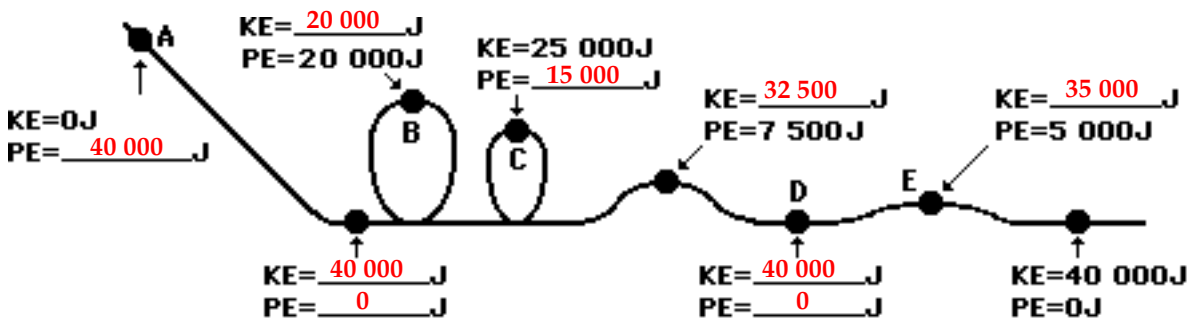
        A = B = C = D = E

8.   Rank the five locations in order of increasing PE (smallest to largest PE). Use < and or = signs between the blanks.

        B < E < D < C < A

9.   Rank the five locations in order of increasing KE (smallest to largest KE). Use < and or = signs between the blanks.
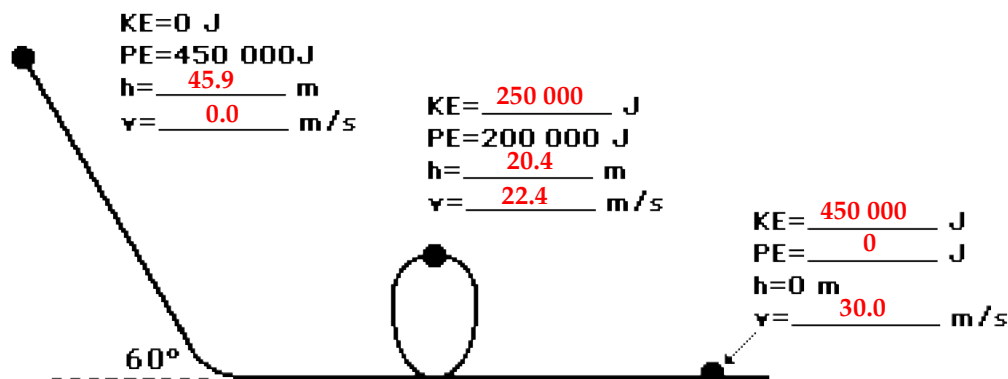
        A < C < D < E < B

10.   Use the law of conservation of energy (assume no friction nor air resistance) to determine the kinetic and potential energy at the various marked positions along the roller coaster track below. Finally, fill in the bars of the bar charts for positions A, B, C, D, and E.

KE=___20 000___ J
PE=20 000 J

KE=0J
PE=___40 000___ J

KE=25 000J
PE=___15 000___ J

KE=___32 500___ J
PE=7 500 J

KE=___35 000___ J
PE=5 000J

KE=___40 000___ J
PE=___0___ J

KE=___40 000___ J
PE=___0___ J

KE=40 000J
PE=0J

| A	B	C	D	E
KE PE TME	KE PE TME	KE PE TME	KE PE TME	KE PE TME

Work, Energy, and Power

11. Use the law of conservation of energy (assume no friction) to fill in the blanks at the various marked positions for a 1000-kg roller coaster car.

KE=0 J
PE=450 000J
h=___45.9___ m
v=___0.0___ m/s

KE=___250 000___ J
PE=200 000 J
h=___20.4___ m
v=___22.4___ m/s

KE=___450 000___ J
PE=___0___ J
h=0 m
v=___30.0___ m/s

60°

The roller coaster begins with 450 000 J of mechanical energy (all in the form of PE). It conserves this energy; so the sum of the KE + PE is 450 000 J at all positions. If KE is known, PE can be calculated (and vice versa). The speeds (v) are related to the KE values by the equation KE = ½•m•v². The heights (h) are related to the PE values by the equation PE = m•g•h.

12. In a physics lab, a 3.0-kg cart was pulled at a constant speed along an inclined plane (with a force parallel to the plane) to a height of 0.500 m. The angle of incline was altered in each consecutive trial. It was found that each angle required the same amount of work to elevate the cart to the same height. Use your understanding of work, energy and dynamics to fill in the following table. (**HINT**: $F_{app} = F_{parallel} = m•g•\sin \Theta$.)
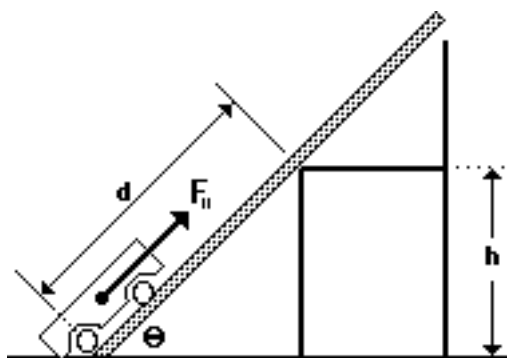
**NOTE**: The Θ value in the work equation is 0° since it is the angle between the F and the d vectors.

	Θ (°)	h (m)	ΔPE (J)	Fparallel (N)	d (m)	Work (J)
a.	15	0.500 m	**14.7 J**	**7.61 N**	**1.93 m**	**14.7 J**
b.	20	0.500 m	**14.7 J**	**10.1 N**	**1.46 m**	**14.7 J**
c.	25	0.500 m	**14.7 J**	**12.4 N**	**1.18 m**	**14.7 J**
d.	35	0.500 m	**14.7 J**	**16.9 N**	**0.872 m**	**14.7 J**
e.	45	0.500 m	**14.7 J**	**20.8 N**	**0.707 m**	**14.7 J**

Show sample calculations below:

For Row a: ΔPE = m•g•Δh = (3.0 kg)•(9.8 N/kg)•(0.500 m) = **14.7 J**
$F_{parallel}$ = m•g•sin Θ = (3.0 kg)•(9.8 N/kg)•sin(15°) = 7.6092 … N = **7.61 N**
d = Δh/(sin Θ) = (0.500 m)/(sin 15°) = 1.9318 ... m = **1.93 m**
W = F•d•(cos Θ) = (7.6092 … N)•(1.9318 ... m)•cos0° = **14.7 J**

13. A wrecking ball is raised above its highest point (State A), possessing 6000 J of PE relative to its lowest location (State B). The wrecking ball strikes a building and comes to a resting position (State C). Determine the kinetic energy of the wrecking ball at state B. __**6000 J**__ Determine the work done on the wrecking ball in going from State B to State C. __**- 6000 J**__


| **State A** | **State B** | **State C** |
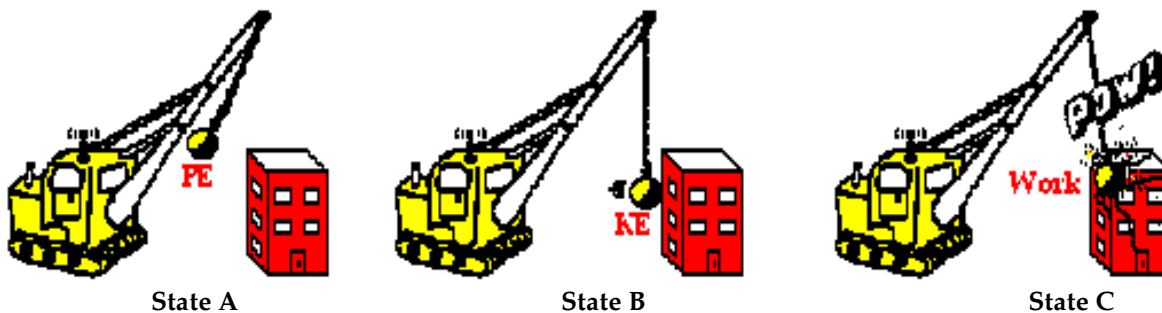
14. Pete Zaria applies a 4.0-N force to a 1.0-kg mug of root beer to accelerate it over a distance of 1.0-meter along the counter top. Determine the work done by Pete on the mug and the mug's final kinetic energy and final velocity. **PSYW**


F = 4.0 N
d = 1.0 m
W = ___**4.0**___ J

Initially,
v = 0 m/s

Finally,
KE = ___**4.0**___ J
v = ___**2.8**___ m/s
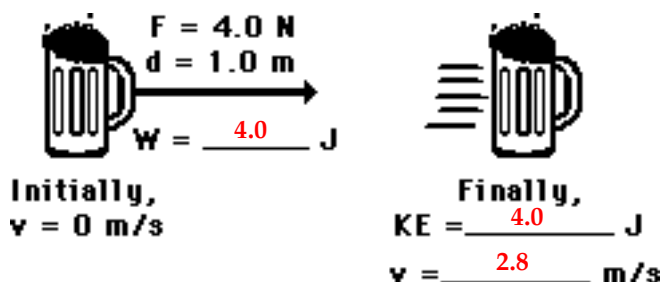
$W = F \cdot d \cdot \cos(\Theta) = (4.0 \text{ N}) \cdot (1.0 \text{ m}) \cdot \cos(0°) = \underline{\textbf{4.0 J}}$
$KE_f = KE_o + W = 0.0 \text{ J} + 4.0 \text{ J} = \underline{\textbf{4.0 J}}$
Since $KE = \frac{1}{2} \cdot m \cdot v^2$, $v = \text{SQRT}(2 \cdot KE/m) = \text{SQRT}(2 \cdot 4.0 \text{ J}/1 \text{ kg}) = \underline{\textbf{2.8 m/s}}$ (2.828 … m/s)

15. A 600-kg roller coaster car (includes passenger mass) is moving at 20.0 m/s. The hydraulic braking system in the track applies an external force to slow the car to a speed of 5.0 m/s over a distance of 20.0 meters. Determine the force that acts upon the car. **PSYW**


v = 20.0 m/s   v = 5.0 m/s

← d = 20.0 m →

F = ___**375**___ N
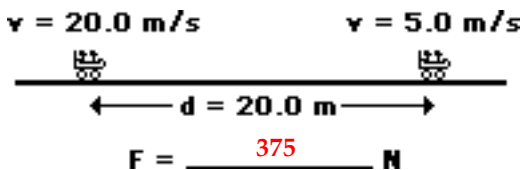
Since $W = \Delta KE = KE_f - KE_o = \frac{1}{2} \cdot m \cdot v_f^2 - \frac{1}{2} \cdot m \cdot v_o^2$ …
$F \cdot d \cdot \cos(\Theta) = \frac{1}{2} \cdot (600 \text{ kg}) \cdot (0.0 \text{ m/s})^2 - \frac{1}{2} \cdot (600 \text{ kg}) \cdot (5.0 \text{ m/s})^2 = -7500 \text{ J}$
$F = (-7500 \text{ J}) / (d \cdot \cos(\Theta)) = (-7500 \text{ J}) / ((20.0 \text{ m}) \cdot \cos(180°)) = \underline{\textbf{375 N}}$

16. Construct work-energy bar charts for problems #14 and #15.

Problem #14   **Problem #15**
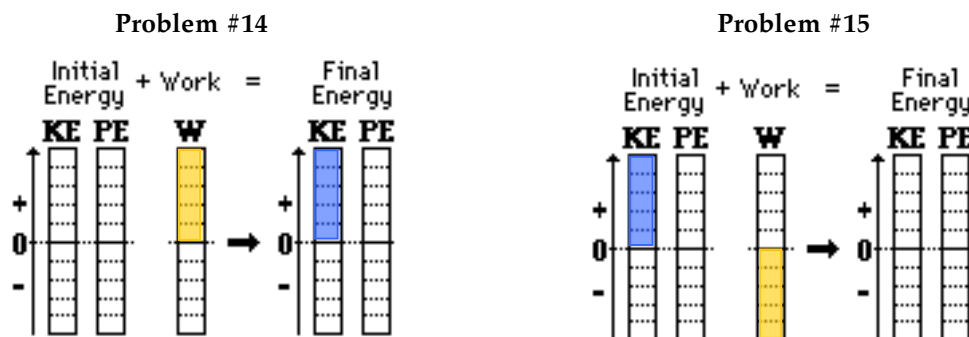
17. Vera is driving her 1000-kg car at a speed of 8.0 m/s. When Vera slams on the brakes, the ground exerts an 8000-N frictional force to bring the car to a stop. Determine the initial kinetic energy of the car, the work done by friction on the car, and the stopping distance of the car. **PSYW**

    Work = ___-32 000___ J

    d = ___4.0___ m

    Initially, v = 8 m/s

    Finally, v = 0 m/s
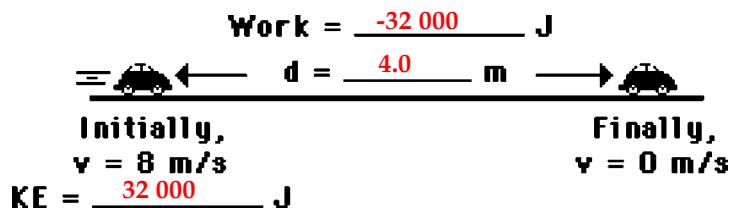
    KE = ___32 000___ J

    The initial KE is ½•(1000 kg)•(8.0 m/s)² = **32000 J**.
    The work done to stop a car with this amount of KE is **-32000 J**.
    The distance over which this work is done is given by the equation W = F•d•cos(Θ). So …
    d = W/(F•d•cos(Θ)) = (-32000 J) / ((8000 N)•cos(180°)) = **4.0 m**

18. If Vera's speed (in question #17) were increased to 24.0 m/s, then what would be the new stopping distance? ___**36.0 m**___ In other words, how many times greater is the stopping distance if the speed is tripled? ___**Nine times greater stopping distance**___ Explain.

    The kinetic energy is proportional to the square of the speed. Three times the speed means nine times the KE and thus nine times as much work that needs to be done. This greater work requires a greater stopping distance - greater by a factor of nine.

19. A 0.750-kg peach can is at rest in a shopping cart at the edge of a hill. A strong wind sets it into motion, sending down a 6.32-meter high hill. The cart hits a tree stump. But the peach can, being in motion, continues in motion until it finally collides with a car. Upon impact, the peach can exerts an average

    KE = 0 J
    PE = ___46.5___ J

    KE = ___46.5___ J

    F·d

    force of 721 N upon the car body. Fill in the blanks and determine the depth of the dent.

    PE$_{initial}$ = m•g•h = (0.750 kg)•(9.8 N/kg)•(6.32 m) = **46.5 J** (46.452 J)
    This initial PE is transformed into KE, giving the can 46.5 J of KE just prior to hitting the car.
    The car does -46.5 J of work on the can to bring it to a stop; this work is supplied by 721 N of force.
    So, d = W/(F•cos(Θ)) = (-46.452 J) / ((721 N)•cos(180°)) **0.0644 m = 6.44 cm**

20. A 56.9 kg sledder descends an 8.21-meter high hill, encountering a friction force of 11.7 N. Fill in the blanks and determine the speed of the sledder after traveling the 31.7 meters to the bottom of the hill.

    KE = 0 J
    PE = ___4580___ J

    W = ___-371___ J

    PE = 0 J
    KE = ___4210___ J

    d

    h$_i$ = 8.21 m
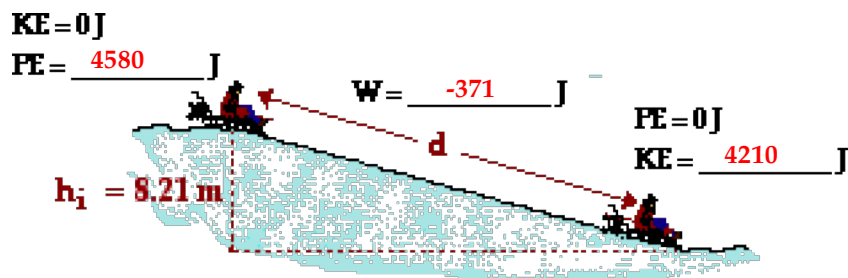
    PE$_{initial}$ = m•g•h = (56.9 kg)•(9.8 N/kg)•(8.21 m) = **4580 J** (4578.06 … J)
    W = F$_{frict}$•d•cos(Θ) = (11.7 N)•(31.7 m)•cos(180°) = **-371 J** (-370.89 J)
    KE$_{final}$ = PE$_{initial}$ + W = 4578.06 … J -370.89 J = **4210 J** (4207.17… J)
    v = SQRT(2•KE/m) = SQRT(2•(4207.17… J)/(56.9 kg)) = SQRT (147.87 … J/kg) = **12.2 m/s**

Work-Energy Calculations

Study **Lesson 2** of the **Work, Energy and Power** chapter at **The Physics Classroom:**

http://www.physicsclassroom.com/Class/energy/u5l2bc.cfm

For the following questions, begin with the work-energy equation, cancel terms, substitute and solve.

1. A glider is gliding through the air at a height of 416 meters with a speed of 45.2 m/s. The glider dives to a height of 278 meters. Determine the glider's new speed.

$$KE_i \quad + \quad PE_i \quad = \quad KE_f \quad + \quad PE_f$$

$$\tfrac{1}{2} \bullet \cancel{m} \bullet (45.2 \text{ m/s})^2 + \cancel{m} \bullet (9.8 \text{ N/kg}) \bullet (416 \text{ m}) = \tfrac{1}{2} \bullet \cancel{m} \bullet v_f^2 + \cancel{m} \bullet (9.8 \text{ N/kg}) \bullet (278 \text{ m})$$

$$\tfrac{1}{2} \bullet v_f^2 = 1021.52 \text{ J} + 4076.8 \text{ J} - 2724.4 \text{ J} = 2373.92 \text{ J}$$

$$\underline{v = 68.9 \text{ m/s}}$$

$v_i = 9.87$ m/s
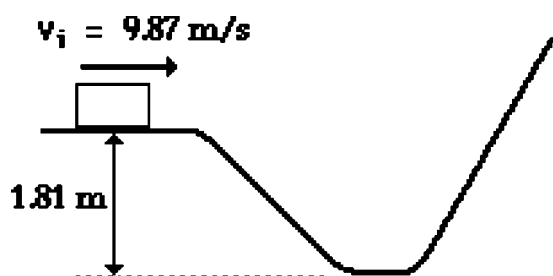
1.81 m

2. A box with mass **m** is sliding along on a friction-free surface at 9.87 m/s at a height of 1.81 m. It travels down the hill and then up another hill.
    a. Find the speed at the bottom of the hill.

$$KE_i \quad + \quad PE_i \quad = \quad KE_f$$

$$\tfrac{1}{2} \bullet \cancel{m} \bullet (9.87 \text{ m/s})^2 + \cancel{m} \bullet (9.8 \text{ N/kg}) \bullet (1.91 \text{ m}) = \tfrac{1}{2} \bullet \cancel{m} \bullet v_f^2$$

$$\tfrac{1}{2} \bullet v_f^2 = 48.708 \ldots \text{ J} + 18.718 \text{ J}$$

$$\underline{v_f = 11.6 \text{ m/s}} \quad (11.612 \ldots \text{ m/s})$$

b. Find the maximum vertical height to which the box will rise on the opposite hill.

$$KE_{bottom} \quad = \quad PE_{highest\ point}$$

$$\tfrac{1}{2} \bullet \cancel{m} \bullet (11.612 \ldots \text{ m/s})^2 = \cancel{m} \bullet (9.8 \text{ N/kg}) \bullet h_f$$

$$67.426 \ldots \text{m}^2/\text{s}^2 = (9.8 \text{ N/kg}) \bullet h_f$$

$$\underline{h_f = 6.88 \text{ m}}$$

3. A 1423-kg car is moving along a level highway with a speed of 26.4 m/s. The driver takes the foot off the accelerator and the car experiences a retarding force of 901-N over a distance of 106 m. Determine the speed of the car after traveling this distance.

$$KE_i \quad + \quad W_{ext} \quad = \quad KE_f$$

$$\tfrac{1}{2} \bullet (1423 \text{ kg}) \bullet (26.4 \text{ m/s})^2 + (901 \text{ N}) \bullet (106 \text{ m}) \bullet \cos(180°) = \tfrac{1}{2} \bullet (1423 \text{ kg}) \bullet v_f^2$$

$$495887.04 \text{ J} - 95506 \text{ J} = (711.5 \text{ kg}) \bullet v_f^2$$

$$\underline{v_f = 23.7 \text{ m/s}}$$

4. A sledder starts from rest atop a 5.0-m high hill (A). She sleds to the bottom and up to the top of the adjacent 3.0-m high hill. How fast is the sledder going at point B? Ignore friction.

$$PE_i \quad = \quad KE_f \quad + \quad PE_f$$

$$\cancel{m} \bullet (9.8 \text{ N/kg}) \bullet (5.0 \text{ m}) = \tfrac{1}{2} \bullet \cancel{m} \bullet v_f^2 + \cancel{m} \bullet (9.8 \text{ N/kg}) \bullet (3.0 \text{ m})$$

$$49.0 \text{ m}^2/\text{s}^2 - 29.4 \text{ m}^2/\text{s}^2 = \tfrac{1}{2} \bullet v_f^2$$

$$\underline{v_f = 6.3 \text{ m/s}} \quad (6.26 \ldots \text{ m/s})$$
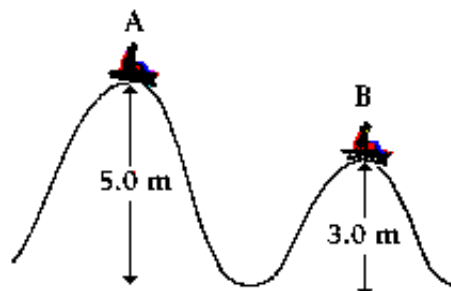
5. A 4768-kg roller coaster train full of riders approaches the loading dock at a speed of 17.1 m/s. It is abruptly decelerated to a speed of 2.2 m/s over a distance of 13.6 m. Determine the retarding force that acts upon the roller coaster cars.

$$KE_i \quad + \quad W_{ext} \quad = \quad KE_f$$
$$\tfrac{1}{2} \bullet (4678 \text{ kg}) \bullet (17.1 \text{ m/s})^2 + F \bullet (13.6 \text{ m}) \bullet \cos(180°) = \tfrac{1}{2} \bullet (4678 \text{ kg}) \bullet (2.2 \text{ m/s})^2$$
$$683946.9 \ldots J - 11320.7 \ldots J = F \bullet (13.6 \text{ m})$$
$$\underline{F = 49500 \text{ N}} \quad (49457.8 \ldots \text{ N})$$

6. A catcher's mitt recoils a distance of 12.9 cm in bringing a 142-gram baseball to a stop. If the applied force is 588 N, then what was the speed of the baseball at the moment of contact with the catcher's mitt?

$$KE_i \quad + \quad W_{ext} \quad = \quad 0 \text{ J}$$
$$\tfrac{1}{2} \bullet (0.142 \text{ kg}) \bullet v_i^2 + (588 \text{ N}) \bullet (0.129 \text{ m}) \bullet \cos(180°) = 0 \text{ J}$$
$$(0.071 \text{ kg}) \bullet v_i^2 = 75.852 \text{ J}$$
$$\underline{v_i = 32.7 \text{ m/s}}$$

7. An unknown force is applied to a 12 kg mass. The force acts at an angle of 30 degrees above the horizontal. Determine the force acting if the force acts for a horizontal displacement of 22 meters and increases the 12 kg mass's speed from 11 m/s to 26 m/s.

$$KE_i \quad + \quad W_{ext} \quad = \quad KE_f$$
$$\tfrac{1}{2} \bullet (12 \text{ kg}) \bullet (11 \text{ m/s})^2 + F \bullet (22 \text{ m}) \bullet \cos(0°) = \tfrac{1}{2} \bullet (12 \text{ kg}) \bullet (26 \text{ m/s})^2$$
$$F \bullet (22 \text{ m}) = 4056 \text{ J} - 726 \text{ J}$$
$$\underline{F = 150 \text{ N}} \quad (151.36 \ldots \text{ N})$$

8. A physics teacher exerts a force upon a 3.29-kg pile of snow to both lift it and set it into motion. The snow leaves the shovel with a speed of 2.94 m/s at a height of 0.562 m. Determine the work done upon the pile of snow.

$$W_{ext} \quad = \quad KE_f \quad + \quad PE_f$$
$$W = \tfrac{1}{2} \bullet (3.29 \text{ kg}) \bullet (2.94 \text{ m/s})^2 + (3.29 \text{ kg}) \bullet (9.8 \text{ N/kg}) \bullet (0.562 \text{ m})$$
$$W = 14.218 \ldots J + 18.120 \ldots J$$
$$\underline{W = 32.3 \text{ J}}$$

9. A 250.-gram cart starts from rest and rolls down an inclined plane from a height of 0.541 m. Determine its speed at a height of 0.127 m above the bottom of the incline.

$$PE_i \quad = \quad KE_f \quad + \quad PE_f$$
$$(0.250 \text{ kg}) \bullet (9.8 \text{ N/kg}) \bullet (0.541 \text{ m}) = \tfrac{1}{2} \bullet (0.250 \text{ kg}) \bullet v_f^2 + (0.250 \text{ kg}) \bullet (9.8 \text{ N/kg}) \bullet (0.127 \text{ m})$$
$$1.325 \ldots J - 0.311 \ldots J = \tfrac{1}{2} \bullet (0.250 \text{ kg}) \bullet v_f^2$$
$$\underline{v_f = 2.85 \text{ m/s}}$$

10. A 4357-kg roller coaster car starts from rest at the top of a 36.5-m high track. Determine the speed of the car at the top of a loop that is 10.8 m high.

$$KPE_i \quad = \quad KE_f \quad + \quad PE_f$$
$$(4357 \text{ kg}) \bullet (9.8 \text{ N/kg}) \bullet (36.5 \text{ m}) = \tfrac{1}{2} \bullet (4357 \text{ kg}) \bullet v_f^2 + (4357 \text{ kg}) \bullet (9.8 \text{ N/kg}) \bullet (10.8 \text{ m})$$
$$1558498.9 \ldots J - 461144.9 \ldots J = \tfrac{1}{2} \bullet (4357 \text{ kg}) \bullet v_f^2$$
$$\underline{v_f = 22.4 \text{ m/s}}$$